Welcome to IB Math: Applications and Interpretation!

This is the summer assignment for rising seniors who were not in my class last year and rising juniors who have already taken algebra 1, algebra 2, and geometry. If you are not taking Math: Applications and Interpretation next year, do the summer assignment for the class you will be taking in September.

This assignment is designed to give you a head start in this course. Here is the link to the google document, which is more interactive. Please complete the lessons thoughtfully, keeping notes, so that we can quickly review them at the beginning of the year. Pace yourself so you don't just rush through them at the end of the summer. We have a lot of material to cover this year so I want you all to be off to a great start.

Copy the notes below. You are welcome and encouraged to restate them in your own words. I embedded some videos in case you want some extra explanations. Keep those notes for yourself. Complete the exercises, showing all of your work. You will turn that work in.

This assignment is due on the first day of school. Make sure you do your best and show your work because this assignment is worth a quiz grade.

1. Number systems
    1.1. Natural numbers - 0, 1, 2, 3, 4, …
        1.1.1. **note:** in the US, natural numbers begin with 1 counting numbers) but in Europe (so in IB) natural numbers begin with 0.
        1.1.2. What Europeans call natural numbers, we call whole numbers in the US but we use the European definition for IB.
        1.1.3. The symbol for natural numbers is $\mathbb{N}$
    1.2. Integers - …, -3, -2, -1, 0, 1, 2, 3, …
        1.2.1. Integers are the natural numbers and their negatives
        1.2.2. The symbol for integers is $\mathbb{Z}$
        1.2.3. $\mathbb{Z}^+$ means the set of positive integers and $\mathbb{Z}^-$ means the negative integers
    1.3. Rational numbers - all the real numbers that can be written as a fraction
        1.3.1. The symbol for rational numbers is $\mathbb{Q}$
        1.3.2. All natural numbers and integers are rational numbers
        1.3.3. All fractions with integers in the numerator and the denominator (except 0 in the denominator) are rational numbers
        1.3.4. All decimals that terminate (end) such as 0.25, 0.1, 0.875, are rational
        1.3.5. All decimals that repeat are rational. Examples: 0.33333, $0.\underline{2}$ (IB uses a dot instead of a bar to show repeating decimals)
    1.4. Irrational numbers
        1.4.1. Real numbers that are not rational are irrational
        1.4.2. Examples include π, $\sqrt{2}$
    1.5. Real numbers
        1.5.1. The symbol for real numbers is $\mathbb{R}$
        1.5.2. Real numbers are all of the numbers on a number line.
        1.5.3. Natural numbers, integers, rational numbers, and irrational numbers are all real numbers.
        1.5.4. Non-real numbers are complex numbers and imaginary numbers. You might have learned about them in algebra 2 but they are not in the IB curriculum.
    1.6. Prime numbers
        1.6.1. Numbers that can only be divided by 1 and the number itself.
        1.6.2. {2, 3, 5, 7, 11, 13, 17, 19, …}
        1.6.3. Composite numbers have other factors. {4, 6, 8, 9, 10, …}
        1.6.4. 1 is neither prime nor composite.

It is important to know the above number families because you might be required to classify numbers while working on some problems.

Number systems examples and exercises:

Explain why:

a any positive integer is also a rational number

b -7 is a rational number

a We can write any positive integer as a fraction where the number itself is the numerator, and the denominator is 1.
For example, $5 = \frac{5}{1}$.
So, all positive integers are rational numbers.

b $-7 = \frac{-7}{1}$, so -7 is rational.

Show that the following are rational numbers:

a 0.47 **b** 0.135

a $0.47 = \frac{47}{100}$, so 0.47 is rational.

b $0.135 = \frac{135}{1000} = \frac{27}{200}$, so 0.135 is rational.

Show that the following recurring decimal numbers are rational:

a 0.777 777 7.... **b** 0.363 636 ....

a Let $x = 0.777\,777\,7....$

$\therefore\ 10x = 7.777\,777\,7....$

$\therefore\ 10x = 7 + 0.777\,777\,....$

$\therefore\ 10x = 7 + x$

$\therefore\ 9x = 7$

$\therefore\ x = \frac{7}{9}$

So, $0.777\,777.... = \frac{7}{9}$, which is rational.

b Let $x = 0.363\,636\,....$

$\therefore\ 100x = 36.363\,636\,....$

$\therefore\ 100x = 36 + 0.363\,636\,....$

$\therefore\ 100x = 36 + x$

$\therefore\ 99x = 36$

$\therefore\ x = \frac{36}{99}$

$\therefore\ x = \frac{4}{11}$

So, $0.363\,636\,.... = \frac{4}{11}$, which is rational.

EXERCISE 1F.1

1 Show that 8 and -11 are rational numbers.

2 Why is $\frac{4}{0}$ not a rational number?

3 Show that the following are rational numbers:

a 0.8 **b** 0.71 **c** 0.45 **d** 0.219 **e** 0.864

4 True or false?

a -136 is a natural number. **b** $\frac{15}{2}$ is a rational number. **c** $\frac{14}{2}$ is not an integer.

5 Show that the following are rational numbers:

a 0.444 444 .... **b** 0.212 121 .... **c** 0.325 325 325 ....

6 On the table, indicate with a tick or cross whether the numbers in the left hand column belong to $\mathbb{Q}$, $\mathbb{Q}^+$, $\mathbb{Z}$, $\mathbb{Z}^+$, or $\mathbb{N}$.

	$\mathbb{Q}$	$\mathbb{Q}^+$	$\mathbb{Z}$	$\mathbb{Z}^+$	$\mathbb{N}$
3					
-2					
1.5					
0					
$-\frac{1}{2}$					

2. SI (Systeme International) units; aka metric units.
   2.1. There are other base units but these are the ones you are expected to know.
   2.2. Prefixes in metric units are based on powers of 10. As you move toward smaller units, multiply the number of units by the appropriate power of 10. As you move toward larger units, divide.

SI base units

Symbol	Name	Quantity
s	second	time
m	metre	length
kg	kilogram	mass

kilo	k	10^3		1 000
hecto	h	10^2		100
deka	da	10^1		10
base unit		10^0		1
deci	d	10^{-1}	1/10	0.1
centi	c	10^{-2}	1/100	0.01
milli	m	10^{-3}	1/1 000	0.001

   2.3. Notice the base unit for mass is kilogram. 1 kg = 1000 grams.
   2.4. You should be able to use these base units in calculations such as finding area, volume, and speed.
   2.5. Many of the formulas will be provided. You will need to know speed = distance/time. Below is a screenshot from the formula booklet provided by IB. You do not need to memorize any formula in the booklet but you need to know how to use them.

Prior learning – SL and HL

Area of a parallelogram	$A = bh$, where b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$, where b is the base, h is the height
Area of a trapezoid	$A = \frac{1}{2}(a+b)h$, where a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Volume of a cuboid	$V = lwh$, where l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$, where r is the radius, h is the height
Volume of prism	$V = Ah$, where A is the area of cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi rh$, where r is the radius, h is the height
Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

[Video on SI units and metric conversions]

SI Examples and Exercises:

a Density is defined as mass per unit volume. Write the SI unit for density.

b A newton is defined as the force which accelerates a mass of 1 kilogram at the rate of 1 metre per second per second. Write down the combination of SI units which defines a newton.

a The unit for mass is kg, and the unit for volume is m^3.
$\therefore$ the unit for density is kg/m^3 or $kg\,m^{-3}$.

b 1 newton = 1 kilogram × 1 metre per second per second = $1\ kg\,m\,s^{-2}$

Smaller or larger multiples of these units are obtained by combining the base unit with a prefix chosen from a progression of powers of 10. The most commonly used are:

nano	n	$10^{-9} = \frac{1}{1\,000\,000\,000}$
micro	μ	$10^{-6} = \frac{1}{1\,000\,000}$
milli	m	$10^{-3} = \frac{1}{1000}$

kilo	k	$10^3 = 1000$
mega	M	$10^6 = 1\,000\,000$
giga	G	$10^9 = 1\,000\,000\,000$

The SI also accepts the prefix "centi" (10^{-2}) which can be used in conjunction with metre, litre, or gram.

When stating the value of a measurement, the prefix chosen should give the value as a number between 0.1 and 1000. Thus, one nautical mile is written as 1.852 km, not 1852 m.

The SI does not allow the use of other units. Imperial units of measurement, as used in the United States for example, are not acceptable in the international system.

Convert:

a 3540 millimetres into metres

b 7.14 kilograms into grams

c 4 hours and 12 minutes into seconds

d 15 knots into kilometres per hour

a
$$1\text{ mm} = 10^{-3}\text{ m}$$
$$\therefore\ 3540\text{ mm} = 3540 \times 10^{-3} = 3.54\text{ m}$$

b
$$1\text{ kg} = 1000\text{ g}$$
$$\therefore\ 7.14\text{ kg} = 7.14 \times 1000 = 7140\text{ g}$$

c
$$1\text{ h} = 3600\text{ s}$$
$$1\text{ min} = 60\text{ s}$$
$$\therefore\ 4\text{ h }12\text{ min} = (4 \times 3600) + (12 \times 60) = 15\,120\text{ s}$$

d
$$1\text{ kn} = 1.852\text{ km}\,h^{-1}$$
$$\therefore\ 15\text{ kn} = 15 \times 1.852 = 27.78\text{ km}\,h^{-1}$$

4 Convert the following:

a 0.025 L into mL
b 26 580 ns into μs
c 45 km into mm
d 5840 kg into t
e 54 kWh into MJ
f 60 $km\,h^{-1}$ into $m\,s^{-1}$
g 0.14 m^2 into mm^2
h 16 $m\,s^{-1}$ into $km\,h^{-1}$
i 36 kn into $km\,h^{-1}$

5 Perform the following conversions, giving your answers in scientific notation:

a 7 L into mL
b 3.8 km into mm
c 9.86 g into kg
d 56 ha into m^2
e 10.8 s into μs
f 258 L into GL

Note: scientific notation is discussed on the next page so review that before completing #5.

3. Rounding rules for IB are very specific and you may lose points for rounding incorrectly or to the incorrect number of places. In general, to round to a certain digit, see if the next digit to the right is ≥ 5. If it is, add one to the digit you are rounding to. If it is < 5, leave the digit as is. [Video on rounding]

 3.1. Decimal approximations - directions sometimes tell you to round to a specified number of decimal places. dp stands for decimal places and that means now many numbers *after* the decimal point. For example, 3.14159 to 1 dp = 3.1, to 2 dp = 3.14, to 3 dp = 3.142 (notice the 1 rounds up because it is followed by a 5)

 3.2. Significant figures - very important to understand. We will cover this in class.

 3.3. Standard form, aka scientific notation. Numbers written in standard form are written as 1 non-zero digit (1, 2, 3, 4, 5, 6, 7, 8, or 9) followed by a decimal point, some more numbers and then times 10 raised to a power. Examples: 345000 = 3.45 x 10^5, 0.02932 = 2.932 x 10^{-2}. [Video]

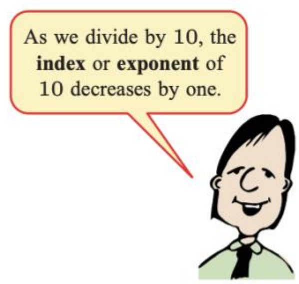
C SCIENTIFIC NOTATION (STANDARD FORM)

Many people doing scientific work deal with very large or very small numbers. To avoid having to write and count lots of zeros, they use **scientific notation** to write numbers.

Observe the pattern:

$$\begin{aligned} 10\,000 &= 10^4 \\ 1000 &= 10^3 \\ 100 &= 10^2 \\ 10 &= 10^1 \\ 1 &= 10^0 \\ \tfrac{1}{10} &= 10^{-1} \\ \tfrac{1}{100} &= 10^{-2} \\ \tfrac{1}{1000} &= 10^{-3} \end{aligned}$$

÷10 −1



We can use this pattern to write very large and very small numbers easily.

For example:

$$\begin{aligned} &5\,000\,000 \\ &= 5 \times 1\,000\,000 \\ &= 5 \times 10^6 \end{aligned} \qquad \text{and} \qquad \begin{aligned} &0.000\,003 \\ &= \tfrac{3}{1\,000\,000} \\ &= 3 \times \tfrac{1}{1\,000\,000} \\ &= 3 \times 10^{-6} \end{aligned}$$

Scientific notation or **standard form** involves writing a given number as a number between 1 and 10, multiplied by a power of 10.

The number is written in the form $a \times 10^k$ where $1 \leqslant a < 10$ and k is an integer.

A number such as 4.62 is already between 1 and 10. We write it in scientific notation as 4.62×10^0 since $10^0 = 1$.

Your calculator is also able to display numbers using scientific notation.

Write in scientific notation:

a 9 448 800 000

b 0.000 000 053 04

a $9\,448\,800\,000$
$= 9.4488 \times 1\,000\,000\,000$
So, $a = 9.4488$ and $k = 9$
The number is 9.4488×10^9.

b $0.000\,000\,053\,04$
$= 5.304 \div 100\,000\,000$
So, $a = 5.304$ and $k = -8$
The number is 5.304×10^{-8}.

Use your calculator to evaluate:

a $870\,000 \times 95\,000\,000$

b $\dfrac{6.2 \times 10^3}{8 \times 10^{12}}$

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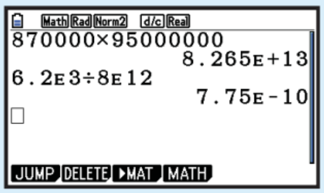

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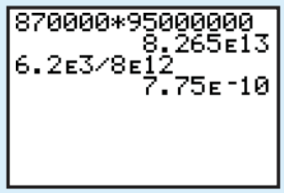

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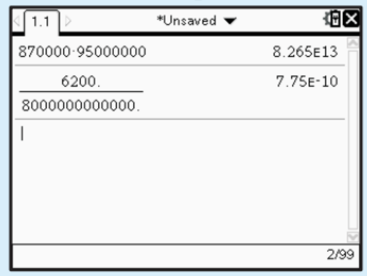

a $870\,000 \times 95\,000\,000 = 8.265 \times 10^{13}$

b $\dfrac{6.2 \times 10^3}{8 \times 10^{12}} = 7.75 \times 10^{-10}$

EXERCISE 2C

1 Which of the following numbers are *not* written in scientific notation?

a 3.7×10^4 **b** 4.2×10^{-7} **c** 0.3×10^5 **d** 21×10^{11}

2 Copy and complete:

$$\begin{aligned} 1000 &= 10^3 \\ 100 &= 10^2 \\ 10 &= \\ 1 &= \\ 0.1 &= \\ 0.01 &= 10^{-2} \\ 0.001 &= \\ 0.0001 &= \end{aligned}$$

3 Write as decimal numbers:

a 8.2×10^4 **b** 3.6×10^1 **c** 8.7×10^0 **d** 4.9×10^2

e 7.8×10^{-3} **f** 5.5×10^{-2} **g** 3.76×10^{-1} **h** 2.02×10^{-3}

4 Write in scientific notation:

a 3900 **b** 17 000 **c** 0.04 **d** 0.000 071
e 85 **f** 6.3 **g** 2 480 000 **h** 0.000 000 108

5 Write these calculator displays in scientific notation:

a 4.5E07 **b** 3.8E-04 **c** 2.1E05 **d** 4.0E-03
e 6.1E03 **f** 1.6E-06 **g** 3.9E04 **h** 6.7E-02

6 Write the numbers displayed in question **5** as decimal numbers.

For example, 3.9E06 is 3.9×10^6
$= 3.900\,000 \times 1\,000\,000$
$= 3\,900\,000$

In an exam it is **not** acceptable to write your answer as a calculator display.

7 Use your calculator to evaluate the following, giving your answer in scientific notation:

a $680\,000 \times 73\,000\,000$ **b** 900^4 **c** $0.0006 \div 15\,000$
d $(0.0007)^3$ **e** $5.3 \times 10^8 \times 6.4 \times 10^5$ **f** $2.6 \times 10^4 \times 3.7 \times 10^{-9}$
g $\dfrac{3.6 \times 10^4}{7.5 \times 10^{11}}$ **h** $\dfrac{4.9 \times 10^{-5}}{1.12 \times 10^6}$

8 Write as a decimal number:

a The estimated population of the world in the year 2020 is 7.4×10^9 people.
b The pressure at the edge of the Earth's thermosphere is about 1.0×10^{-7} Pa.
c The diameter of the Milky Way is 1.4×10^5 light years.
d The mass of a proton is about 1.67×10^{-27} kg.

9 Express the following in scientific notation:

a The Jurassic period lasted about 54 400 000 years.
b The ball bearing in a pen nib has diameter 0.003 m.
c There are about 311 900 000 different 5-card poker hands which can be dealt.
d The wavelength of blue light is about 0.000 000 47 m.

10 Last year a peanut farmer produced 6×10^4 kg of peanuts. If the peanuts weighed an average of 8×10^{-4} kg, how many peanuts did the farm produce? Give your answer in scientific notation.

4. Other numerical concepts
   4.1. Absolute value - the distance from a number to zero on a number line. Distance is never a negative number so the absolute value of a number is always positive or zero.
      4.1.1. The symbol for absolute value is 2 bars. | | with a number between them.
      4.1.2. $|2| = 2$; $|-5| = 5$; $|0| = 0$
      4.1.3. To find absolute value on a TI-84 calculator, press the MATH key and go over to NUM. You will see 1: ABS(. Press this and then type the number.
   4.2. Operations with signed numbers (positive and negative)
      4.2.1. Addition: if the numbers have the same sign, add them together and keep the sign. If the numbers have different signs, subtract the smaller from the larger and give the difference the sign of the larger. $3 + 5 = 8$; $-4 + (-5) = -9$; $4 + (-6) = -2$; $-5 + 6 = 1$
      4.2.2. Subtraction: keep the first number as is, change to addition, change the sign of the second (keep, change, change) then add the numbers. $-5 - 7 = -5 + (-7) = -12$; $-3 - (-6) = -3 + 6 = 3$
      4.2.3. Multiplication and division: 2 positives or 2 negatives equals positive. One positive and one negative equals negative. $-3 \times (-2) = 6$; $-10 / 2 = -5$.
   4.3. Operations with fractions: video
      4.3.1. Addition and subtraction require common denominators. Then add or subtract the numerators and keep the denominator. $\frac{1}{2} + \frac{1}{3} = 3/6 + 2/6 = \frac{5}{6}$.
      4.3.2. Multiplication: multiply the numerators together and multiply the denominators together. $\frac{3}{4} \times 5/7 = 15/28$
      4.3.3. Division: keep the first number as is, change to multiplication, find the reciprocal of the second fraction (keep, change, flip) $\frac{2}{5} \div 3/8 = \frac{2}{5} \times 8/3 = 16/15$
      4.3.4. In IB, you do not have to simplify fractions.
   4.4. Order of operations
      4.4.1. PEMDAS: parentheses (all grouping symbols), exponents, multiplication and division in order from left to right, addition and subtraction in order from left to right.
      4.4.2. I see a lot of mistakes in order of operations so it is advisable to know how to type equations into your calculator.
      4.4.3. Parenthese stand for all grouping symbols (fraction bars, in an exponent together, …) They can be confusing so see comment above this one.

Video on order of operations with integers   Video on order of operations with fractions

Evaluate: $2 \times (3 \times 6 - 4) + 7$

$$\begin{aligned} & 2 \times (3 \times 6 - 4) + 7 \\ &= 2 \times (18 - 4) + 7 && \{\text{inside brackets, multiply}\} \\ &= 2 \times 14 + 7 && \{\text{evaluate expression in brackets}\} \\ &= 28 + 7 && \{\text{multiplication next}\} \\ &= 35 && \{\text{addition last}\} \end{aligned}$$

2 Evaluate the following:

a $(11 - 6) \times 3$

d $4 \times (6 - 2)$

g $2 + 3 \times (7 - 2)$

j $4 \div (3 - 1) + 6$

m $2 \times (3 - 4) + (7 - 1)$

Evaluate: $5 + [13 - (8 \div 4)]$

$$\begin{aligned} & 5 + [13 - (8 \div 4)] \\ &= 5 + [13 - 2] && \{\text{innermost brackets first}\} \\ &= 5 + 11 && \{\text{remaining brackets next}\} \\ &= 16 && \{\text{addition last}\} \end{aligned}$$

3 Simplify:

a $3 \times [2 + (7 - 5)]$

b $3 + [2 \times (7 - 5)]$

d $[14 \div (2 + 5)] \times 3$

e $3 + [32 \div (2 + 6)] \div 2$

Example 13 Self Tutor

Evaluate: $\dfrac{16 - (4 - 2)}{14 \div (3 + 4)}$

$$\begin{aligned} & \frac{16 - (4 - 2)}{14 \div (3 + 4)} \\ &= \frac{16 - 2}{14 \div 7} && \{\text{brackets first}\} \\ &= \frac{14}{2} && \{\text{evaluate numerator, denominator}\} \\ &= 7 && \{\text{do the division}\} \end{aligned}$$

For a fraction we evaluate the numerator and denominator separately, then perform the division.

4 Simplify:

a $\dfrac{19 - 3}{2}$

5 Simplify:

a $3 + 5^2$

d $(13 - 4) \div 3^2$

6 Simplify:

a $3 \times -2 + 18$

d $[3 - (-2 + 7)] + 4$

g $-6 \times (2 - 7)$

j $-52 \div (6 - 19)$

Use your calculator to simplify $\dfrac{27 + 13}{5 \times 4}$.

We first write the fraction as $\dfrac{(27 + 13)}{(5 \times 4)}$.

So, $\dfrac{27 + 13}{5 \times 4} = 2$.

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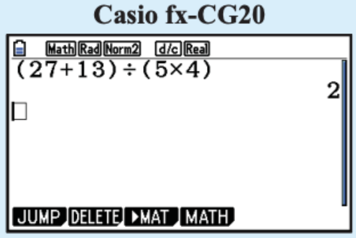

1 Use your calculator to simplify:

a $6 \times 8 - 18 \div (2 + 4)$

c $5 + (2 \times 10 - 5) - 6$

e $(2 \times 3 - 4) + (33 \div 11 + 5)$

g $(50 \div 5 + 6) - (8 \times 2 - 4)$

i $(7 - 3 \times 2) \div (8 \div 4 - 1)$

k $\dfrac{27 - (18 \div 3) + 3}{3 \times 4}$