Welcome to IB Math: Applications and Interpretation!

This is the summer assignment for rising seniors who were not in my class last year and rising juniors who have already taken algebra 1, algebra 2, and geometry. If you are not taking Math: Applications and Interpretation next year, do the summer assignment for the class you will be taking in September. If you are taking Applications and Interpretation and are not yet in the googleclassroom, please join IB Summer Math 2023, **wa2ykxq**.

This assignment is designed to give you a head start in this course. Please complete the lessons thoughtfully, keeping notes, so that we can quickly review them at the beginning of the year. Pace yourself so you don't just rush through them at the end of the summer. We have a lot of material to cover this year so I want you all to be off to a great start.

On the following pages you will find information on 3 topics: number families, international units of measure, and numerical concepts. Each section consists of 4 parts: notes in outline form, worked examples in blue boxes, exercises for you to complete, and videos. Copy the notes into your IB math notebook. You are welcome and encouraged to restate them in your own words. Keep those notes for yourself. Complete the exercises, showing all of your work. You will turn that work in at the beginning of school next year.

The last 6 problems require a calculator. You are strongly encouraged to have a graphing calculator of your own. The pace of your end of the year IB exam is quick; you need to be proficient in use of a graphing calculator. If you don't have a calculator, you may use desmos.com for a graphing calculator or desmos.com/scientific. If you need to purchase a calculator, check for sales (Target had the lowest prices I could find last summer), ask friends who recently graduated, or check eBay or other resale sites. I recommend you have either a TI-84 or TI-Nspire.

This assignment is due on the first day of school. Make sure you do your best and show your work because this assignment is worth your first quiz grade.

1) Number systems
    a) Natural numbers - 0, 1, 2, 3, 4, …
        i) **note:** in the US, natural numbers begin with 1 counting numbers) but in Europe (so in IB) natural numbers begin with 0.
        ii) What Europeans call natural numbers, we call whole numbers in the US but we use the European definition for IB.
        iii) The symbol for natural numbers is $\mathbb{N}$
    b) Integers - …, -3, -2, -1, 0, 1, 2, 3, …
        i) Integers are the natural numbers and their negatives
        ii) The symbol for integers is $\mathbb{Z}$
        iii) $\mathbb{Z}^+$ means the set of positive integers and $\mathbb{Z}^-$ means the negative integers
    c) Rational numbers - all the real numbers that can be written as a fraction
        i) The symbol for rational numbers is $\mathbb{Q}$
        ii) All natural numbers and integers are rational numbers
        iii) All fractions with integers in the numerator and the denominator (except 0 in the denominator) are rational numbers
        iv) All decimals that terminate (end) such as 0.25, 0.1, 0.875, are rational
        v) All decimals that repeat are rational. Examples: 0.33333, $0.\overline{2}$ (IB uses a dot instead of a bar to show repeating decimals)
    d) Irrational numbers
        i) Real numbers that are not rational are irrational
        ii) Examples include π, √2
    e) Real numbers
        i) The symbol for real numbers is $\mathbb{R}$
        ii) Real numbers are all of the numbers on a number line.
        iii) Natural numbers, integers, rational numbers, and irrational numbers are all real numbers.
        iv) Non-real numbers are complex numbers and imaginary numbers. You might have learned about them in algebra 2 but they are not in the IB curriculum.
    f) Prime numbers
        i) Numbers that can only be divided by 1 and the number itself.
        ii) {2, 3, 5, 7, 11, 13, 17, 19, …}
        iii) Composite numbers have other factors. {4, 6, 8, 9, 10, …}
        iv) 1 is neither prime nor composite.

It is important to know the above number families because you might be required to classify numbers while working on some problems.

Number systems examples and exercises:

Explain why:

a any positive integer is also a rational number

b -7 is a rational number

a We can write any positive integer as a fraction where the number itself is the numerator, and the denominator is 1.

For example, $5 = \frac{5}{1}$.

So, all positive integers are rational numbers.

b $-7 = \frac{-7}{1}$, so -7 is rational.

Show that the following are rational numbers:

a 0.47 **b** 0.135

a $0.47 = \frac{47}{100}$, so 0.47 is rational.

b $0.135 = \frac{135}{1000} = \frac{27}{200}$, so 0.135 is rational.

Show that the following recurring decimal numbers are rational:

a 0.777 777 7.... **b** 0.363 636 ....

a Let $x = 0.777\,777\,7....$

$\therefore\ 10x = 7.777\,777\,7....$

$\therefore\ 10x = 7 + 0.777\,777....$

$\therefore\ 10x = 7 + x$

$\therefore\ 9x = 7$

$\therefore\ x = \frac{7}{9}$

So, $0.777\,777.... = \frac{7}{9}$, which is rational.

b Let $x = 0.363\,636....$

$\therefore\ 100x = 36.363\,636....$

$\therefore\ 100x = 36 + 0.363\,636....$

$\therefore\ 100x = 36 + x$

$\therefore\ 99x = 36$

$\therefore\ x = \frac{36}{99}$

$\therefore\ x = \frac{4}{11}$

So, $0.363\,636.... = \frac{4}{11}$, which is rational.

EXERCISE 1F.1

1 Show that 8 and -11 are rational numbers.

2 Why is $\frac{4}{0}$ not a rational number?

3 Show that the following are rational numbers:

a 0.8 **b** 0.71 **c** 0.45 **d** 0.219 **e** 0.864

4 True or false?

a -136 is a natural number. **b** $\frac{15}{2}$ is a rational number. **c** $\frac{14}{2}$ is not an integer.

5 Show that the following are rational numbers:

a 0.444 444 .... **b** 0.212 121 .... **c** 0.325 325 325 ....

6 On the table, indicate with a tick or cross whether the numbers in the left hand column belong to $\mathbb{Q}$, $\mathbb{Q}^+$, $\mathbb{Z}$, $\mathbb{Z}^+$, or $\mathbb{N}$.

	$\mathbb{Q}$	$\mathbb{Q}^+$	$\mathbb{Z}$	$\mathbb{Z}^+$	$\mathbb{N}$
3					
-2					
1.5					
0					
$-\frac{1}{2}$					

Note for #6 - each number might be classified as more than one type of number.

2) SI (Systeme International) units; aka metric units.
   a) There are other base units but these are the ones you are expected to know.
   b) Prefixes in metric units are based on powers of 10. As you move toward smaller units, multiply the number of units by the appropriate power of 10. As you move toward larger units, divide.

SI base units

Symbol	Name	Quantity
s	second	time
m	metre	length
kg	kilogram	mass

kilo	k	10^3		1 000
hecto	h	10^2		100
deka	da	10^1		10
base unit		10^0		1
deci	d	10^{-1}	1/10	0.1
centi	c	10^{-2}	1/100	0.01
milli	m	10^{-3}	1/1 000	0.001

   c) Notice the base unit for mass is kilogram. 1 kg = 1000 grams. That is because kg is more commonly used. However, when converting units, think of gram (g) as the base unit.
   d) You should be able to use these base units in calculations such as finding area, volume, and speed.
   e) Many of the formulas will be provided. You will need to memorize speed = distance/time (think miles per hour). IB provides a formula booklet containing most of the formulas you will use this year. You do not need to memorize any formula in the booklet but you need to know how to use them.

Video on SI units and metric conversions

SI Examples and Exercises:

a Density is defined as mass per unit volume. Write the SI unit for density.

b A newton is defined as the force which accelerates a mass of 1 kilogram at the rate of 1 metre per second per second. Write down the combination of SI units which defines a newton.

a The unit for mass is kg, and the unit for volume is m^3.

$\therefore$ the unit for density is kg/m^3 or $kg\,m^{-3}$.

b 1 newton = 1 kilogram × 1 metre per second per second = $1\ kg\,m\,s^{-2}$

Smaller or larger multiples of these units are obtained by combining the base unit with a prefix chosen from a progression of powers of 10. The most commonly used are:

nano	n	$10^{-9} = \frac{1}{1\,000\,000\,000}$
micro	μ	$10^{-6} = \frac{1}{1\,000\,000}$
milli	m	$10^{-3} = \frac{1}{1000}$

kilo	k	$10^3 = 1000$
mega	M	$10^6 = 1\,000\,000$
giga	G	$10^9 = 1\,000\,000\,000$

The SI also accepts the prefix "centi" (10^{-2}) which can be used in conjunction with metre, litre, or gram.

When stating the value of a measurement, the prefix chosen should give the value as a number between 0.1 and 1000. Thus, one nautical mile is written as 1.852 km, not 1852 m.

The SI does not allow the use of other units. Imperial units of measurement, as used in the United States for example, are not acceptable in the international system.

Convert:

a 3540 millimetres into metres

b 7.14 kilograms into grams

c 4 hours and 12 minutes into seconds

d 15 knots into kilometres per hour

a
$$1\ mm = 10^{-3}\ m$$
$$\therefore\ 3540\ mm = 3540 \times 10^{-3} = 3.54\ m$$

b
$$1\ kg = 1000\ g$$
$$\therefore\ 7.14\ kg = 7.14 \times 1000 = 7140\ g$$

c
$$1\ h = 3600\ s$$
$$1\ min = 60\ s$$
$$\therefore\ 4\ h\ 12\ min = (4 \times 3600) + (12 \times 60) = 15\,120\ s$$

d
$$1\ kn = 1.852\ km\,h^{-1}$$
$$\therefore\ 15\ kn = 15 \times 1.852 = 27.78\ km\,h^{-1}$$

Convert the following:

a 0.025 L into mL

b 26 580 ns into μs

c 45 km into mm

d 5840 kg into t

e 54 kWh into MJ

f 60 $km\,h^{-1}$ into $m\,s^{-1}$

g 0.14 m^2 into mm^2

h 16 $m\,s^{-1}$ into $km\,h^{-1}$

i 36 kn into $km\,h^{-1}$

Hint for h: convert with 2 factors: convert the distance and then convert the time. Note that the time is in the denominator. Ex: 23 cm s^{-1} to m h^{-1}: $\frac{23cm}{s} \times \frac{1\ m}{100cm} \times \frac{3600s}{1\ h} = 828\ mh^{-1}$

For i: kn (knots): 1 kn = 1.852 km h^{-1}

3) Other numerical concepts
   a) Absolute value - the distance from a number to zero on a number line. Distance is never a negative number so the absolute value of a number is always positive or zero.
      i) The symbol for absolute value is 2 bars. | | with a number between them.
      ii) |2| = 2; |-5| = 5; |0| = 0
      iii) To find absolute value on a TI-84 calculator, press the MATH key and go over to NUM. You will see 1: ABS(. Press this and then type the number.
   b) Operations with signed numbers (positive and negative)
      i) Addition: if the numbers have the same sign, add them together and keep the sign. If the numbers have different signs, subtract the smaller from the larger and give the difference the sign of the larger. 3 + 5 = 8; -4 + (-5) = -9; 4 + (-6) = -2; -5 + 6 = 1
      ii) Subtraction: keep the first number as is, change to addition, change the sign of the second (keep, change, change) then add the numbers. -5 - 7 = -5 + (-7) = -12; -3 - (-6) = -3 + 6 = 3
      iii) Multiplication and division: 2 positives or 2 negatives equals positive. One positive and one negative equals negative. -3 x (-2) = 6; -10 / 2 = -5.
   c) Operations with fractions: video
      i) Addition and subtraction require common denominators. Then add or subtract the numerators and keep the denominator. ½ + ⅓ = 3/6 + 2/6 = ⅚.
      ii) Multiplication: multiply the numerators together and multiply the denominators together. ¾ x 5/7 = 15/28
      iii) Division: keep the first number as is, change to multiplication, find the reciprocal of the second fraction (keep, change, flip) ⅖ ➗3/8 = ⅖ x 8/3 = 16/15
      iv) In IB, you do not have to simplify fractions.
   d) Order of operations
      i) PEMDAS: parentheses (all grouping symbols), exponents, multiplication and division in order from left to right, addition and subtraction in order from left to right.
      ii) I see a lot of mistakes in order of operations so it is advisable to know how to type equations into your calculator.
      iii) Parenthese stand for all grouping symbols (fraction bars, in an exponent together, …) They can be confusing so see comment above this one.

Video on order of operations with integers   Video on order of operations with fractions

Evaluate: $35 - 10 \div 2 \times 5 + 3$

$35 - 10 \div 2 \times 5 + 3$
$= 35 - 5 \times 5 + 3$ {division and multiplication working from left}
$= 35 - 25 + 3$
$= 10 + 3$ {subtraction and addition working from left}
$= 13$

1 Evaluate the following:

a $6 - 3 + 4$

d $3 \times 2 - 1$

g $9 - 6 \div 3$

j $3 + 9 \div 3 - 2$

Evaluate: $2 \times (3 \times 6 - 4) + 7$

$$\begin{aligned} & 2 \times (3 \times 6 - 4) + 7 \\ &= 2 \times (18 - 4) + 7 && \{\text{inside brackets, multiply}\} \\ &= 2 \times 14 + 7 && \{\text{evaluate expression in brackets}\} \\ &= 28 + 7 && \{\text{multiplication next}\} \\ &= 35 && \{\text{addition last}\} \end{aligned}$$

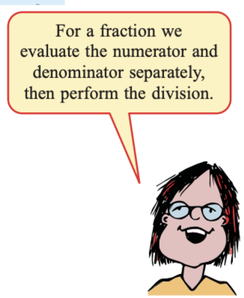
Evaluate: $5 + [13 - (8 \div 4)]$

$$\begin{aligned} & 5 + [13 - (8 \div 4)] \\ &= 5 + [13 - 2] && \{\text{innermost brackets first}\} \end{aligned}$$

Example 13 **Self Tutor**

Evaluate: $\dfrac{16 - (4 - 2)}{14 \div (3 + 4)}$

$$\begin{aligned} & \frac{16 - (4 - 2)}{14 \div (3 + 4)} \\ &= \frac{16 - 2}{14 \div 7} && \{\text{brackets first}\} \\ &= \frac{14}{2} && \{\text{evaluate numerator, denominator}\} \\ &= 7 && \{\text{do the division}\} \end{aligned}$$

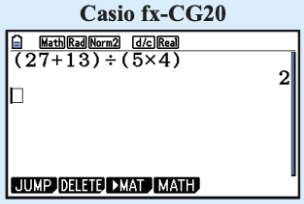


Use your calculator to simplify $\dfrac{27 + 13}{5 \times 4}$.

We first write the fraction as $\dfrac{(27 + 13)}{(5 \times 4)}$.

So, $\dfrac{27 + 13}{5 \times 4} = 2$.

Casio fx-CG20



2 Evaluate the following:

- **a** $(11 - 6) \times 3$
- **d** $4 \times (6 - 2)$
- **g** $2 + 3 \times (7 - 2)$
- **j** $4 \div (3 - 1) + 6$
- **m** $2 \times (3 - 4) + (7 - 1)$

3 Simplify:

- **a** $3 \times [2 + (7 - 5)]$
- **b** $3 + [2 \times (7 - 5)]$
- **d** $[14 \div (2 + 5)] \times 3$
- **e** $3 + [32 \div (2 + 6)] \div 2$

4 Simplify:

- **a** $\dfrac{19 - 3}{2}$

5 Simplify:

- **a** $3 + 5^2$
- **d** $(13 - 4) \div 3^2$

6 Simplify:

- **a** $3 \times -2 + 18$
- **d** $[3 - (-2 + 7)] + 4$
- **g** $-6 \times (2 - 7)$
- **j** $-52 \div (6 - 19)$

1 Use your calculator to simplify:

- **a** $6 \times 8 - 18 \div (2 + 4)$
- **c** $5 + (2 \times 10 - 5) - 6$
- **e** $(2 \times 3 - 4) + (33 \div 11 + 5)$
- **g** $(50 \div 5 + 6) - (8 \times 2 - 4)$
- **i** $(7 - 3 \times 2) \div (8 \div 4 - 1)$
- **k** $\dfrac{27 - (18 \div 3) + 3}{3 \times 4}$

Tips for solving long expressions with multiple operations on a calculator:

1. If possible, type the entire expression in as it is written. This will be the most accurate method.
2. For fractions, see if your calculator makes fractions (google it). If it doesn't, encase both the numerator and the denominator in parentheses as shown on the screenshot of a casio right above this.
3. If you cannot type the entire expression in at once, carefully follow the order of operations.